

## BRIEF PAPER

# Analysis of Optical Power Splitter with Resonator Structure Constructed by Two-Dimensional MDM Plasmonic Waveguide

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**SUMMARY** An efficient optical power splitter constructed by a metal-dielectric-metal plasmonic waveguide with a resonator structure has been analyzed. The method of solution is the finite difference time domain (FD-TD) method with the piecewise linear recursive convolution (PLRC) method. The resonator structure consists of input/output waveguides and a narrow waveguide with a T-junction. The power splitter with the resonator structure is expressed by an equivalent transmission-line circuit. We can find that the transmittance and reflectance calculated by the FD-TD method and the equivalent circuit are matched when the difference in width between the input/output waveguides and the narrow waveguide is small. It is also shown that the transmission wavelength can be adjusted by changing the narrow waveguide lengths that satisfy the impedance matching condition in the equivalent circuit.

**key words:** plasmonic waveguide, optical power splitter, optical resonator, FD-TD method, PLRC method, transmission-line circuit

## 1. Introduction

The plasmonic waveguides are one of the candidates for the realization of highly integrated optical circuits because they can overcome the diffraction limit of light [1]–[3]. In particular, since the planar metal-dielectric-metal (MDM) plasmonic waveguide can be fabricated using existing CMOS processes, it is expected to further miniaturize optical circuit devices based on silicon photonics technology [4]. To integrate highly optical devices, there is a demand to develop highly efficient fundamental components, such as bent waveguides, optical power splitters, and so on. When there is even a slight reflected wave at the operating wavelength from an optical component, a Fabry-Pérot resonance will occur between the optical components, making it impossible to obtain the desired wavelength characteristics of the system. One of the methods to suppress reflected waves at the operating wavelengths from the optical component is utilizing resonators. The resonators can suppress reflections at the operating wavelengths and improve transmission efficiency. In addition to suppressing reflections, the resonators are widely applied in filters [5], [6], multi/demultiplexers [7], [8], splitters [9], [10], sensors [11], [12], and so on.

In this paper, we analyze a characteristics of two-dimensional optical power splitter constructed by MDM

plasmonic waveguides with a resonator which improves the transmission efficiency by resonant tunneling [13]. The resonator is composed of input/output waveguides and a narrow waveguide with a T-junction. The reflected wave from the resonator is eliminated when the impedances of the input and the output ports are matched. In the structure, the quality factor of the resonator can be adjusted by changing the width of the narrow waveguide, so it is expected that fine adjustment of the transmission band can be easier than the resonator using gaps [14]. However, the transmission characteristics greatly depend on the width and the length of the waveguides. Therefore, in order to understand the structure parameter dependence of the transmission characteristics, we express the power splitter as an equivalent transmission-line circuit. The transmission characteristics of the transmission-line circuit are compared with the result of the finite-difference time-domain (FD-TD) method with the piecewise linear recursive convolution (PLRC) method [15]. The numerical results show that the transmission characteristics of the transmission-line circuit agree with those of the FD-TD method when the difference in width between the input/output waveguides and the narrow waveguide is small. The results also show that the transmission wavelength and bandwidth can be controlled by adjusting the length and width of the narrow waveguide.

## 2. Formulation of Problems

We consider a two-dimensional MDM plasmonic waveguides constructed by gold and air as shown in Fig. 1 (a). The MDM plasmonic waveguide used here has cladding layers of gold which is low-loss for infrared light, and a waveguide layer of air whose refractive index is 1.0. We connect a narrow waveguide with widths  $W_n = W_4 = W_5 = W_6$  and lengths  $L_4, L_5, L_6$  to the input/output waveguide with a width  $W$ . The resonator can be realized by utilizing impedance mismatch at the junction between the narrow waveguide and the input/output waveguide. The gold is dispersive media and it has a negative permittivity for infrared and visible light [16], [17]. The method of solution is the FD-TD method with the PLRC method [15] for dispersive media. The computational zone is surrounded by the perfectly matched layers (PMLs) [18] which are used as the absorbing boundary condition.

The dielectric constant of gold in the infrared wavelengths is estimated by the Drude model [16] as follows:

Manuscript received July 10, 2023.

Manuscript revised November 15, 2023.

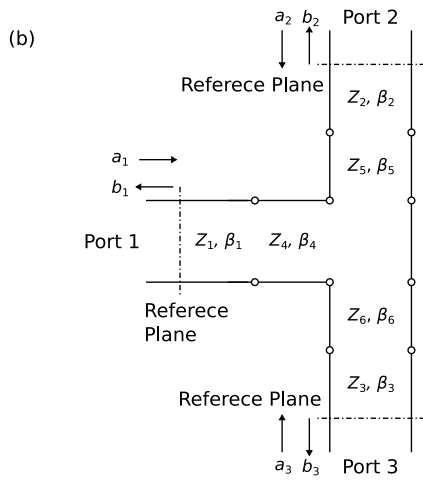
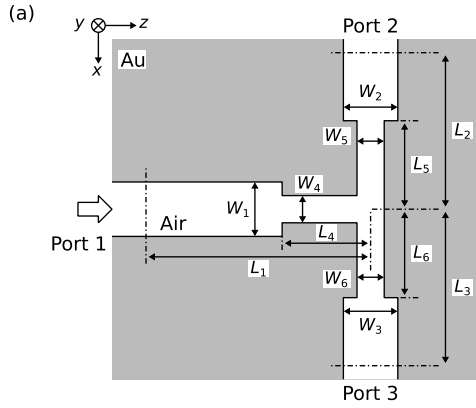
Manuscript publicized December 7, 2023.

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DOI: 10.1587/transle.2023ECS6008

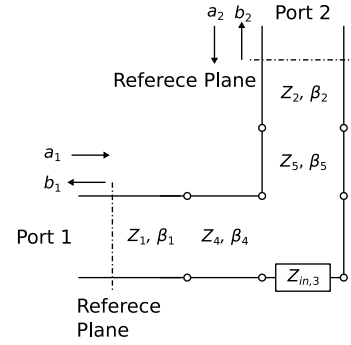


**Fig. 1** 2D MDM plasmonic optical power splitter with resonator structure. (a) waveguide structure, (b) equivalent transmission-line circuit.

$$\varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + j\frac{\omega}{\tau}} \quad (1)$$

where  $\omega_p = 1.38 \times 10^{16}$  Hz and  $\tau = 9.3 \times 10^{-15}$  s are plasma frequency and relaxation time, respectively. The dielectric constant for infrared light has dispersibility and its imaginary part causes the optical loss. To investigate the power loss due to the waveguide structure excluding material loss, we employ the real part of the Drude model as the dielectric constant of the gold in this study. The propagation loss due to material loss for a waveguide width of 100 nm at a wavelength of 1.55  $\mu\text{m}$  is estimated at approximately 0.3 dB/ $\mu\text{m}$ . The polarization of the incident wave is transverse magnetic (TM) mode ( $H_y, E_x, E_z$ ) because the MDM plasmonic waveguide has no surface plasmon polariton (SPP) mode for transverse electric (TE) mode. In this study, the input/output waveguides with width of  $W = W_1 = W_2 = W_3 = 100$  nm are employed to operate in the single-mode regime. The reference planes are placed at  $L_1 = L_2 = L_3 = 1 \mu\text{m}$  from the center of T-junction. The grid cell sizes of  $\delta = \delta_x = \delta_z = 2.5$  nm, and the time step of  $c\delta t = 0.625\delta$  are used.

The power splitter constructed by the MDM plasmonic waveguide can be expressed by a 3-port transmission-line cir-



**Fig. 2** Equivalent 2-port transmission-line circuit of the optical power splitter for analyzing transmission characteristics of port 2.

cuit [14], [19] as shown in Fig. 1 (b) where  $Z_i$  and  $\beta_i$  ( $i = 1$  to 6) are the characteristic impedance and propagation constant for waveguide  $i$ , respectively. The mode field profile in the core can be regarded as constant when the incident wavelength  $\lambda$  is sufficiently large with respect to the waveguide width  $W$ . In addition, the penetration depth of the evanescent field in the cladding becomes small when the absolute value of the dielectric constant of the metal in the cladding  $|\varepsilon_{\text{metal}}|$  is sufficiently larger than that of the dielectric in the core  $\varepsilon_{\text{diel}}$ . When these two conditions ( $W \ll \lambda$  and  $|\varepsilon_{\text{metal}}| \gg \varepsilon_{\text{diel}}$ ) are satisfied, the TM mode propagating in the MDM plasmonic waveguide can be approximated to the TEM mode propagating in the parallel plate waveguide [1], [20], which has the characteristic impedance as follows:

$$Z_i = \frac{\beta_i}{\omega \varepsilon_{\text{diel}}} W_i \quad (2)$$

where  $\omega$  is angular frequency of the incident wave. The propagation constant  $\beta_i$  is calculated by numerically solving the dispersion eigenvalue equation for  $\text{TM}_0$  mode of the two-dimensional MDM plasmonic waveguide [21]. The 3-port transmission-line circuit can be expressed by an equivalent 2-port transmission-line circuit as shown in Fig. 2.  $Z_{in,i}$  appeared in the figure is the input impedance of port  $i$  seen at the T-junction [22] expressed by

$$Z_{in,i} = Z_{i+3} \frac{Z_i + jZ_{i+3} \tan(\beta_{i+3}L_{i+3})}{Z_{i+3} + jZ_i \tan(\beta_{i+3}L_{i+3})}, \quad i = 1 \text{ to } 3 \quad (3)$$

The transmission characteristics of port 2 can be analyzed by using the 2-port circuit. The transmission characteristics of port 3 can be also analyzed similarly. The incident and reflected waves  $a_i$  and  $b_i$ , where  $i$  is the port number, are related by the transmission matrix ( $T$ -matrix)  $\mathbf{T}$  [22] as follows:

$$\begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \mathbf{T}^{(i)} \begin{pmatrix} a_i \\ b_i \end{pmatrix}, \quad \mathbf{T}^{(i)} = \begin{pmatrix} T_{11}^{(i)} & T_{12}^{(i)} \\ T_{21}^{(i)} & T_{22}^{(i)} \end{pmatrix}, \quad i = 2, 3 \quad (4)$$

The  $T$ -matrix of the 2-port network is calculated by

$$\mathbf{T}^{(i)} = \Phi_1 \mathbf{M}_{1,4} \Phi_4 \mathbf{T}_B^{(i)} \Phi_{i+3} \mathbf{M}_{i+3,i} \Phi_i, \quad i = 2, 3 \quad (5)$$

where  $\Phi_i$  and  $\mathbf{M}_{i,j}$  are the  $T$ -matrices described the phase

shift in the waveguide  $i$  and the junction between waveguide  $i$  and  $j$ , respectively, which are expressed by

$$\Phi_i = \begin{pmatrix} e^{-j\beta_i L_i} & 0 \\ 0 & e^{j\beta_i L_i} \end{pmatrix} \quad (6)$$

$$\mathbf{M}_{i,j} = \frac{1}{2\sqrt{Z_i Z_j}} \begin{pmatrix} Z_i + Z_j & -Z_i + Z_j \\ -Z_i + Z_j & Z_i + Z_j \end{pmatrix} \quad (7)$$

The matrix  $\mathbf{T}_B^{(i)}$  ( $i = 2, 3$ ) describes the two-port network with port impedance  $Z_4$  and  $Z_5$  or  $Z_6$ , which consists of a single impedance  $Z_{in,3}$  or  $Z_{in,2}$  connected in series, respectively, as follows:

$$\mathbf{T}_B^{(i)} = \frac{1}{2\sqrt{Z_4 Z_{i+3}}} \times \begin{pmatrix} Z_4 + Z_{i+3} - Z_{in,j} & -Z_4 + Z_{i+3} + Z_{in,j} \\ -Z_4 + Z_{i+3} - Z_{in,j} & Z_4 + Z_{i+3} + Z_{in,j} \end{pmatrix} \quad (8)$$

where  $(i, j) = (2, 3)$  or  $(3, 2)$ . From the definition of Eq. (4), the reflection coefficient to port 1 ( $S_{11}$ ) and the transmission coefficient to port 2 ( $S_{21}$ ) or port 3 ( $S_{31}$ ) is calculated by

$$S_{11} = -\frac{T_{12}^{(2)}}{T_{22}^{(2)}} = -\frac{T_{12}^{(3)}}{T_{22}^{(3)}}, S_{21} = \frac{1}{T_{22}^{(2)}}, S_{31} = \frac{1}{T_{22}^{(3)}} \quad (9)$$

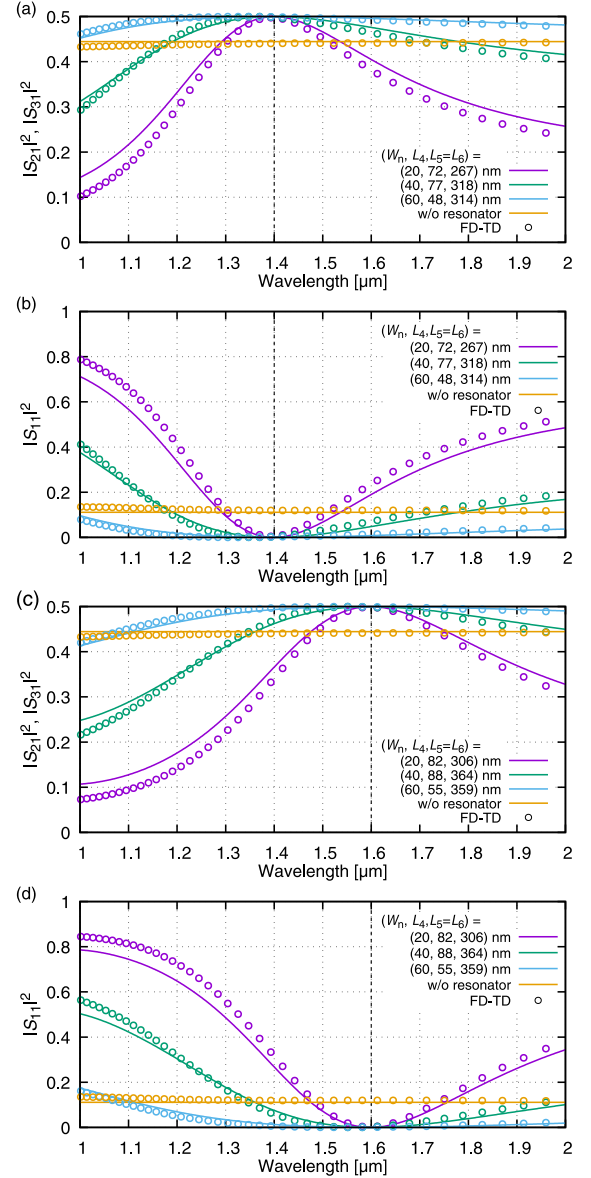
The reflected power to the input port can be minimized when the impedance matching condition between the input port (port 1) and output ports (ports 2 and 3) is satisfied as follows:

$$Z_{in,1} = (Z_{in,2} + Z_{in,3})^* \quad (10)$$

where  $*$  denotes the complex conjugate. The condition can be satisfied by choosing the combination of the length of the narrow waveguides  $L_{i+3}$  because  $Z_{in,i}$  depends on  $L_{i+3}$ .

### 3. Numerical Results

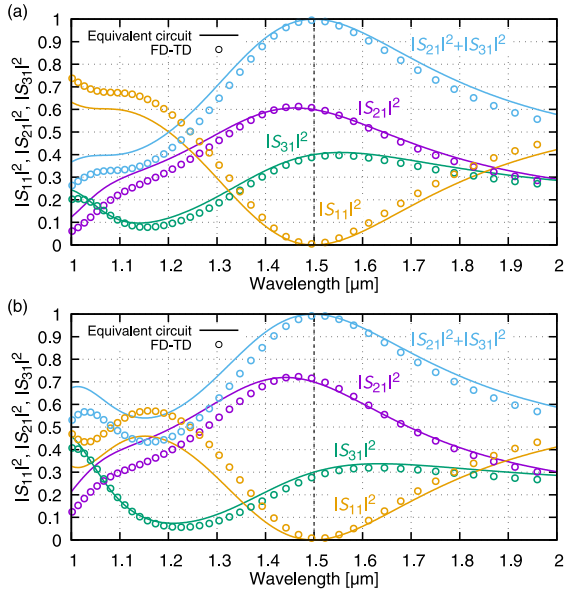
Figure 3 shows the optical power transmission and reflection spectra of the optical power splitter with resonator structure for several narrow waveguide widths  $W_n$ . The lengths of the narrow waveguide  $L_{i+3}$  are chosen in order that the resonant wavelength is  $1.4 \mu\text{m}$  or  $1.6 \mu\text{m}$ . To split the optical power evenly, the lengths of  $L_5$  and  $L_6$  are set to be equal. For comparison, the results for that without a resonator structure are also shown in the figure. It can be confirmed that the reflected optical power can be minimized and all the input optical power can be distributed to the output waveguides at the arbitrary resonant wavelength by choosing the appropriate combination of the length of the narrow waveguides  $L_4$  and  $L_5 = L_6$ . The bandwidth can be broadened by decreasing the difference in width between the input/output waveguides and the narrow waveguide  $\Delta W = W - W_n$ , because the reflection at the junction becomes small, resulting in a decrease in the quality factor of the resonator. As a result, the optical power transmittance and reflectance are improved in wide bandwidth compared to that of the optical power splitter without the resonator structure. It is also



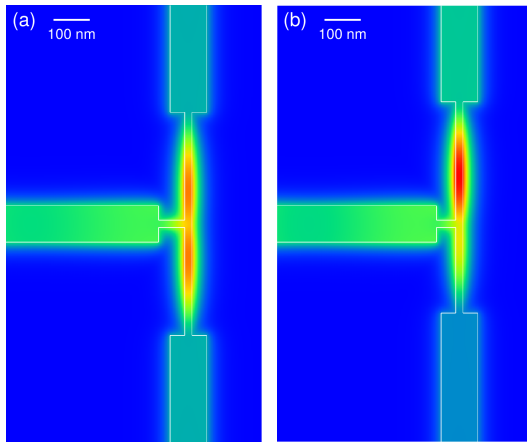
**Fig. 3** Optical power (a)(c) transmission and (b)(d) reflection spectra of the optical power splitter with resonator structure for several narrow waveguide widths  $W_n$ . The combination of the narrow waveguide lengths  $L_4, L_5 (= L_6)$  are chosen in order that the resonant wavelength is (a)(b)  $1.4 \mu\text{m}$  or (c)(d)  $1.6 \mu\text{m}$ . The solid lines and circles denote the results of the equivalent circuit and the FD-TD method, respectively.

found that the larger the difference in waveguide width  $\Delta W$  at the junction, the larger the difference between the results of the FD-TD method and the transmission-line circuit. This may be due to the difference in mode profiles at the junction, which is not considered in the transmission-line circuit. If  $\Delta W$  is large, the reflection due to the mismatch of the mode profile becomes large. On the other hand, if  $\Delta W$  is small and there are few reflections due to the mismatch of the mode profile, it is considered that the transmission-line circuit gives a good approximation.

We next analyze the asymmetrical optical power splitter that has different length of the narrow waveguides  $L_5$  and  $L_6$ .



**Fig. 4** Optical power transmission and reflection spectra of the asymmetric optical power splitter with  $W_n = 20$  nm. The combination of the narrow waveguide lengths (a)  $(L_4, L_5, L_6) = (74, 310, 264)$  nm and (b)  $(63, 335, 245)$  nm are chosen in order that the reflected power is minimized at the wavelength of  $1.5 \mu\text{m}$ . The solid lines and circles denote the results of the equivalent circuit and the FD-TD method, respectively.



**Fig. 5** Magnetic field intensity  $|H_y|$  at the resonant wavelength of the optical power splitter with  $W_n = 20$  nm. The combination of the narrow waveguide lengths and the incident wavelength  $\lambda$  are (a)  $(L_4, L_5, L_6) = (82, 306)$  nm,  $\lambda = 1.6 \mu\text{m}$ , and (b)  $(L_4, L_5, L_6) = (63, 335, 245)$  nm,  $\lambda = 1.5 \mu\text{m}$ , respectively.

Figure 4 shows the optical power transmission and reflection spectra of the asymmetric optical power splitter for the combination of the narrow waveguide lengths  $(L_4, L_5, L_6)$  that are chosen in order that the reflected power is minimized at the wavelength of  $1.5 \mu\text{m}$ . The results show that the distribution ratio can be changed while the reflected power is suppressed by using the combination of the narrow waveguide lengths that satisfies the impedance matching condition.

The distribution of magnetic field intensities  $|H_y|$  of the symmetric and the asymmetric optical power splitter at the resonant wavelength with the narrow waveguide width  $W_n =$

$20$  nm are shown in Fig. 5. We can find that the symmetric and the asymmetric distributions can be confirmed. We can also see that the reflection waves are suppressed because there are few standing wave patterns in the input waveguide for both structures.

#### 4. Conclusion

We have analyzed an optical power splitter with a resonator, which consists of input/output waveguides and narrow waveguides with a T-junction, constructed by a two-dimensional MDM plasmonic waveguide by using the FD-TD method with the PLRC method. We have expressed the power splitter as an equivalent transmission-line circuit. From the results, we have found that the transmittance and reflectance of the equivalent circuit agreed with those of the FD-TD method when the difference in width between the input/output waveguides and the narrow waveguide is small. The numerical results showed that the optical power is entirely distributed to the output waveguides at the resonant wavelength and the bandwidth can be broadened by decreasing the difference in width between the input/output waveguides and the narrow waveguide. We have also shown that the distribution ratio can be changed while the reflected power is suppressed by taking into account the impedance matching condition. From the results, we have found that the equivalent circuit can facilitate the estimation of the structure parameters for arbitrary resonant wavelength.

The investigation of the effects of material loss will be a future work.

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